It has been concluded from histological and physiological evidence that the suprarenal bodies of Teleostean fishes consist solely of cortex.* The physiologically active material is wanting, as in cortical substance elsewhere, and it would be interesting to determine the presence or absence of the chromogen. Unfortunately we have been unable so far to obtain sufficient material for chemical examination. The same remarks apply to the suprarenal bodies of the Ganoids.

For the purpose of comparison, we have chemically tested an extract made from the suprarenal glands of the frog. Six good-sized animals were killed and the suprarenals snipped off from the kidney with scissors. Although there was a considerable admixture of kidney substance with the material thus obtained, the weight in a moist state only amounted to 0·13 gram. This was treated in the way described above for the Elasmobranch material, and gave the chromogen reactions in a perfectly definite manner.

"Memoir on the Integration of Partial Differential Equations of the Second Order in Three Independent Variables, when an Intermediary Integral does not exist in general."

By A. R. Forsyth, F.R.S., Sadlerian Professor in the University of Cambridge.

Received November 23,—Read December 16, 1897.

(Abstract.)

The general feature of most of the methods of integration of any partial differential equation is the construction of an appropriate subsidiary system and the establishment of the proper relations between integrals of this system and the solution of the original equation. Methods, which in this sense may be called complete, are possessed for partial differential equations of the first order in one dependent variable and any number of independent variables; for certain classes of equations of the first order in two independent variables and a number of dependent variables; and for equations of the second (and higher) orders in one dependent and two independent variables. The present memoir discusses the theory of partial differential equations of the second order in one dependent and three independent variables; and the method adopted is seen, without difficulty, to be applicable to equations which involve more than three independent variables and which can be of order higher than the second. The reason why equations of the type considered are taken to be such as do not possess an intermediary integral, that is, a

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* Vincent, *loc. cit.*

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differential equation of lower order in virtue of which the original equation is satisfied, is that the other equations, which do possess an intermediary integral, are a class apart and have been considered elsewhere.*

In order to solve a given equation, a system of subsidiary equations is constructed; and the system is made up of two parts. One of these parts is a set of simultaneous partial differential equations in two independent variables and a number of dependent variables, this number being one more than the number of the equations. An integral equivalent of this part accordingly contains an undetermined quantity. The other of the parts is a set of equations in a single independent variable; it appears that the set of equations in the second part can be consistently satisfied by a determination of the unknown quantity emerging from the first part.

In particular, the equations represented by

$$F(a, b, c, f, g, h, l, m, n, v, x, y, z) = 0,$$

where x, y, z are the independent variables, v is the dependent variable, l, m, n are its first derivatives, and a, b, c, f, g, h are its second derivatives, are found to divide themselves into two distinct classes according as the equation

$$\xi^2\frac{\partial F}{\partial a}+\eta^2\frac{\partial F}{\partial b}+\zeta^2\frac{\partial F}{\partial c}+2\xi\eta\frac{\partial F}{\partial h}+2\eta\zeta\frac{\partial F}{\partial g}+2\eta\zeta\frac{\partial F}{\partial f}=0$$

is resolvable or is not resolvable into two equations linear in ξ, η, ζ. When this equation, called the characteristic invariant on account of an invariantive property which it is proved to possess, is resolvable into two linear equations, the process of integration of the subsidiary equations is much simplified.

The first of the three sections, into which the paper is divided, deals with the general theory, and indicates a method whereby subsidiary equations for an equation $F = 0$ of any degree in the derivatives of the second order can be constructed. If integrable combinations of the subsidiary system are not obtainable, an extension of the method shows how equations of higher order (when obtainable) can be deduced and associated with the given equation.

The second of the three sections deals with those equations of which the characteristic invariant is resolvable; and some examples are given, alike of equations for which the integration of the initial subsidiary system is possible, and of equations for which the extended method must be used.

The third of the three sections deals with those equations of which the characteristic invariant is irresolvable. Of such equations the most interesting examples (from the point of view of application to mathematical physics) are the equations

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* 'Camb. Phil. Trans.,' vol. 16, pp. 191–218.

$$\nabla^2 v = 0,$$
$$\nabla^2 v = -\kappa^2 v,$$

or, with the notation of the memoir,

$$a+b+c = 0,$$
$$a+b+c = -\kappa^2 v;$$

and the theory is applied to these equations in detail. Solutions, which are believed to be new, are obtained for both of them; each solution involves two explicit arbitrary functional forms, and the argument of each of these arbitrary functions itself involves an arbitrary element; but in each case the solution is not that of the widest possible generality which the equation is known to possess. To quote one result: a solution of the equation

$$a+b+c = 0$$

can be stated as follows:—

Let p, q, r denote three arbitrary functions of u subject solely to the condition

$$p^2+q^2+r^2 = 0;$$

let u be determined as a function of x, y, z by means of the equation

$$au = xp(u)+yq(u)+zr(u),$$

where a is a constant, and let v denote

$$\mathrm{H}(u)+\frac{\mathrm{G}(u)}{a-xp'(u)-yq'(u)-zr'(u)},$$

where G and H are distinct arbitrary functions: then v satisfies the equation

$$a+b+c = \nabla^2 v = 0.$$

"On the Biology of *Stereum hirsutum*, Fr." By H. MARSHALL WARD, D.Sc., F.R.S., Professor of Botany in the University of Cambridge. Received November 23,—Read December 16, 1897.

(Abstract.)

The author has cultivated the mycelium of this fungus obtained from spores, on sterilised wood, and after several months the cultures developed yellow bosses which proved to be the hymenophores bearing the basidia. This fungus has not hitherto been made to produce